Velocity of Isolated Particles Along a Pipe in Stratified Gas–Liquid Flow

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The average velocity of isolated grains of sand was experimentally measured in smooth stratified flow in slightly declined pipes. Isolated particles in smooth stratified flow behave similarly to isolated particles propelled by both hydraulic conveying and intermittent gas/liquid flow. In all three cases, particle velocity is linear with respect to the average liquid velocity of the flow (or the average fluid velocity in the slug body for intermittent flow) and has a gradient of approximately one. The data in stratified flow are successfully correlated dimensionlessly (Eq. 7). The correlation is extrapolated to zero particle velocity to estimate the conditions required to ensure sand transport in a flowline in smooth stratified flow. The experimental results suggest that particle velocity is strongly governed by the size of a particle relative to the depth of the viscous sublayer at the pipe wall. If a particle is larger than the viscous sublayer, it is exposed to more coherent turbulent structures and therefore experiences a greater drag.

Introduction

The behavior of solids in multiphase subsea flowlines has become an issue of increasing industrial interest over the past decade. Settled sand in a flowline can cause decreased flow from a well and an environment conducive to electrochemical corrosion. It is, therefore, essential that sand be either excluded from the flowline by means of gravel traps at the well-head or managed in the flowline so that deposition is prevented. Tanner (2000) has highlighted the difficulty of maintaining equipment at the well-head and is therefore inclined to believe that sand management is preferable to sand exclusion.

Stevenson and Thorpe (1999) showed that sand holdup in a typical flowline is likely to be around 5–40 ppm. Therefore, it is more pertinent to consider the behavior of isolated particles (that is, very low solids concentration) rather than moving beds of sand. Most previous work in the related field of hydraulic conveying of solids has considered relatively high solids holdup, as it is uneconomical to transport solids (coal for instance) in a hydraulic pipeline at low holdup. Experiments on the transport of isolated particles in hydraulic conveying and horizontal intermittent flow are discussed in Stevenson et al. (2001a,b). The approach used in the aforementioned two articles will be used in this work to analyze data for the movement of particles by a smooth stratified flow which is another flow regime commonly encountered in subsea oil flowlines. Experimental results will be presented and the data will be used to develop dimensionless correlations for particle velocity and the threshold of particle motion in stratified flow.

Experimental Procedure

The rig used is shown in Figure 1. It consists of a transparent perspex pipe of 12 m in length and interchangeable internal diameters of 40 mm and 70 mm. The pipeline, being hinged at the upstream end, can be moved to any angle up to, or down from, 3° to the horizontal. The pipe is supplied with liquid from a tank by means of a centrifugal pump with a maximum flow rate of 0.001 m^3s^{-1}. The test pipe is fabricated from 800 mm long flanged perspex sections and the pipe is gasketted at the joints. Liquids used were water and a solution of Rheovis CR2, an associative colloidal thickener (Allied Colloids Ltd.) with Newtonian rheology. The viscosity of the Rheovis CR2 solution was measured using a capillary viscometer. Air is available from a main supplying at 1.4 barg and regulated to 0.5 barg before being introduced at the upstream end of the test pipe. The air supply line enters the test pipe perpendicularly from above. In practice, the strati-

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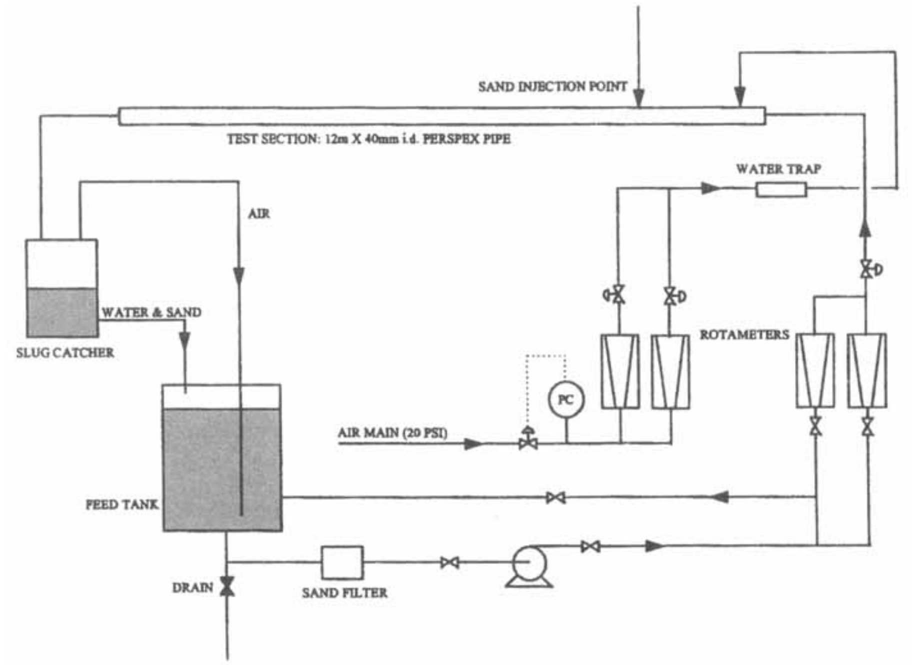

Figure 1. Experimental rig.

fied flow tests were conducted at zero gas rate for the reasons discussed below.

Two calibrated rotameters are installed in parallel for both the air and liquid enabling determination of flow rates. As the viscosity of the liquid is changed, the change in drag coefficient of the rotameter float is small so there is no need to recalibrate at each viscosity (McCabe et al. 1993).

Pulses of approximately ten particles are introduced to the test pipe via a modified squeezable plastic wash bottle containing sand and water 3 meters downstream of the gas/liquid confluence. The pulse of particles was seen to diffuse on entering the stream and little interparticle interaction was observed as they progressed downstream.

At the downstream end of the test section is a drum, known as the slug-catcher, which is used to separate the gas from the liquid and reduce pressure fluctuations in the test section. Such fluctuations are caused by the simultaneous passage of air and water back to the liquid tank. The liquid and gas separate in the slug-catcher and travel back to the pipe in dedicated lines. The air return line terminates 1.8 m below the liquid surface in the tank, thus ensuring an approximately constant pressure of 0.17 barg in the slug-catcher.

Particle velocity was measured by taking transit times of particles between starts and finish lines marked on the pipe. The observer would "spot" a particle, chosen at random, and follow it down the test section, timing the transit between start and finish lines by means of a stopwatch. This was repeated several times for each gas/liquid flow rate. Sand was timed over a distance of 5.72 m close to the outlet end of the pipe.

Three grades of sand, one size of Olivine (magnesium iron silicate) and one size of lead shot were used in the tests. Some properties of the particles are shown in Table 1. The particle densities were measured by the displacement of water. It should be noted that the density of elemental lead at 300 K is reported as 11,330 kg$\cdot$m^{-3} by Perry and Green (1984).

Table 1. Properties of Particles Used in the Experimental Program

Material	Sieving Dia.	Density	Circularity
Coarse sand	1,000–1,180 μm	2,540 kg$\cdot$m^{-3}	0.883
Medium sand	425–600 μm	2,540 kg$\cdot$m^{-3}	0.913
Fine sand	150–300 μm	2,540 kg$\cdot$m^{-3}	0.910
Olivine	707–841 μm	3,300 kg$\cdot$m^{-3}	0.871
Lead shot	1,000–1,180 μm	11,200 kg$\cdot$m^{-3}	0.920

The circularity C of a shape is defined as

$$C = \frac{p_e}{p} \tag{1}$$

where p is the perimeter of a two-dimensional (2-D) projection of an object and p_e is the perimeter of the circle that has the same projected area. Digital photographs (see Figures 2–6) were taken of a sample of each of the particles and their attributes were measured using particle analysis software *ScionImage*.

It is thought that a systematic study of the effect of a particle's shape on its transport behavior has never been carried out. However, it seems likely (Stevenson et al. 2001a) that shape will have an effect, for example, spheres are expected to be easier to transport than cubes. The *circularity* of the lead shot is similar to that of the sands despite clearly, if subjectively, being much more rounded (see Figure 2). It is

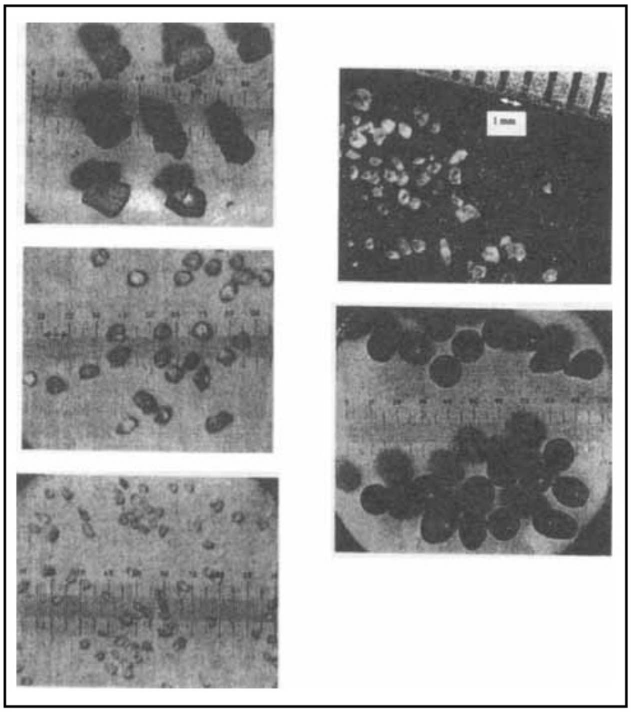

Figure 2. Magnified (a) coarse sand, (b) medium sand, (c) fine sand, (d) Olivine (magnesium iron silicate), and (e) lead shot.

intuitive that a more rounded particle, all other things being equal, is easier to move than a more angular one. This argument demonstrates the inadequacy of a one parameter description of the shape of 3-D object. Further work is necessary on the effect of particle shape on its mobility. The effect that the shape has on the mobility of a particle will be discussed in a later section of this article. The coefficient of static friction between sand grains and the pipe was found to be 0.55. The main effect of greater surface roughness would be to create a modest increase in the coefficient of static friction. The effect of changing the coefficient of static friction is not addressed in this article, although it is an explicit factor in the theory of incipient motion of particles from pipe walls developed by Stevenson et al. (2001c).

Calculating the Average Liquid Velocity in Downhill Smooth Stratified Flow

Stevenson et al. (2001a) noticed that the velocity of grains of sand were linear with respect to average liquid velocity in hydraulic conveying. Stevenson et al. (2001b) showed that sand velocity was linear with respect to the average velocity of liquid within the slug body in intermittent flow. We therefore anticipated that it would be the average velocity of the liquid stratum w, that is, a measure of the velocity of the liquid as it flows past the sand particles, that governs particle behavior in smooth stratified flow rather than the liquid superficial velocity j_f. It is therefore necessary to predict w as a function of j_f.

Agrawal et al. (1973) equated the pressure gradient in the gas and liquid for stratified flow in order to determine liquid holdup and, therefore, the average liquid velocity, in the lower stratum. The pressure gradient was computed by calculating the liquid side wall shear stress, gas side shear stress, and interfacial shear stress. They assumed the following formulations to describe the shear stresses: $\tau_{wf} = 0.5C_f^f \rho_f w^2$, $\tau_{wg} = 0.5C_f^g \rho_g u_g^2$ and $\tau_i = 0.5C_f^i \rho_g (u_g - w)^2$. For turbulent flow, Taitel and Dukler (1976) suggested that the standard Blasius formulation should be used for the liquid and gas friction factors: $C_f^f = 0.046Re_f^{-0.2}$ and $C_f^g = 0.046\text{Re}_g^{-0.2}$, where $\text{Re}_f = wD_f/\nu_f$ and $\text{Re}_g = u_g D_g/\nu_g$. D_f and D_g are the liquid- and gas-phase hydraulic diameters calculated as suggested by Agrawal et al. (1973): $D_f = 4A_f/P_f$ and $D_g = 4A_g/(P_f + P_i)$. A_f and A_g are the cross sectional areas of the liquid and gas strata, respectively, P_f and P_g are the wetted perimeters of the liquid and gas strata, respectively, and P_i is the chordal width of the interface between gas and liquid strata. Based on limited experimental data, Taitel and Dukler suggested that, for smooth stratified flow, the interfacial friction factor C_j^i is approximately equal to the gas-phase friction factor. The flow can then be calculated by finding the liquid holdup that gives equal gas and liquid pressure gradients.

Andreussi and Persen (1987) performed experiments on smooth stratified flows in a 0.05 m I.D. acrylic pipe declined at 0.65° and 2.10° using liquids of viscosity 1 cP and 2.8 cP. They found that Taitel and Dukler's method almost exactly predicted the experimental liquid holdup when the gas rate was zero and performed very well when the actual gas velocity was greater than 5 m/s (no experiments were performed at non-zero gas superficial velocities of less than 5 m/s). A number of "improved" formulations of friction factors have been proposed in the past 20 years (Kowalski, 1987). However, because Andreussi and Persen showed the success of the method of Taitel and Dukler in predicting the flow in a test rig similar to the one used in the current work, the liquid velocities presented in this work will be calculated by the method of Taitel and Dukler. Values of calculated w as a function of j_f are shown in Table 2 for a 2° declined pipe of internal diameter 0.07 m.

Stevenson (2001) shows that interfacial shear stress is negligible compared to the shear exerted on the liquid stratum at the pipe wall for gas superficial velocities of up to 5 m/s in a

Table 2. Calculated Velocity of Liquid Stratum as a Function of Liquid Superficial Velocity for a Hydraulically Smooth Pipe of 0.07 m ID Declined at 2° to the Horizontal

Liquid Superficial Velocity j_f (m/s)	Liquid Stratum Velocity w (m/s)
0.02	0.56
0.04	0.69
0.06	0.78
0.08	0.84
0.10	0.90
0.12	0.96
0.14	1.00
0.16	1.04
0.18	1.08
0.20	1.11
0.22	1.14
0.24	1.17
0.26	1.20
0.28	1.22
0.30	1.25

pipe of 0.07 m I.D. The experiments reported herein were therefore conducted at zero gas superficial velocity. Therefore, the interfacial shear stress is approximately zero and the velocity of the liquid stratum w may be calculated by equating forces due to the wall shear stress on the liquid stratum and the force due to the weight of the fluid resolved axially with respect to the pipe.

Solids Transport in Stratified Flow

The velocity of isolated particles in liquid-gas smooth stratified flow in both 0.04 m and 0.07 m pipes was measured. The test section was declined at 1° and 2° to the horizontal. Individual sand grains were visually tracked over a distance of 5.72 m.

It was found to be meaningless to measure an average sand velocity in the 0.04 m, one degree declined pipe in the range $0.3 < j_f < 0.7$ m/s as the particle moved only sporadically down the test section. The sand came to a halt at certain locations in the pipe, especially at the gasketted joints where it is possible small flaws in the pipeline were offering a sheltered or more stable environment to the sand grains. The phenomenon may also be due to the fact that the coefficient of friction between the rubber gasket and the particles is higher than that between the pipe wall and the particles. Miniature dunes were observed. Initially approximately 80% of encroaching grains were captured, with the remainder being able to circumnavigate the obstruction. The mini dune eventually settled at about thirty grains in size. The majority of encroaching grains were then deflected into more turbulent regions of flow and were able to proceed downstream, while the original grains in the mini dunes seemed to remain in perpetuity.

Duen formation was neither observed when the 0.04 m pipe was declined to 2°, nor at either 1° or 2° in the 0.07 m pipe, with the sand being able to freely saltate down the test section. The speed of the water, being greater in the 0.07 m pipe than the 0.04 m pipe at the same declination and liquid superficial velocity, appeared to be enough to overcome any force that arose due to pipe imperfections.

The velocity of sand of mean sieving diameter of 1.09 mm was measured in stratified flow for fluid viscosities in the range 1–4.8 cP in the 0.04 m pipe. The results (Figure 3) show that, within experimental error, particle velocity increases linearly with average water velocity.

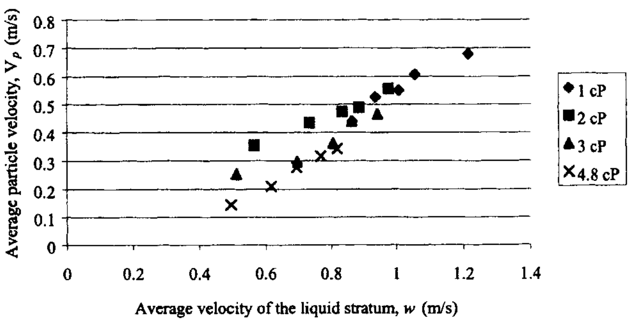

Figure 3. Average velocity of coarse sand at various liquid velocities in a 0.04 m ID pipe declined at 2° to the horizontal.

Particle velocity generally appears to decrease with increasing liquid velocity. This is consistent with the observations of Stevenson et al. (2001a,b) for hydraulic conveying and intermittent flow at low particle loading.

It can be seen from Figure 4 that the average particle velocity increases linearly with the velocity of the liquid stratum in the 0.07 m pipe, replicating the observations made on experiments in the 0.04 m pipe. It is seen that larger particles travel more quickly than smaller ones. A dependence on particle density is also observed. The olivine and the lead shot travels less quickly than sand of about the same size. These observations mirror the observations of Stevenson et al. (2001a,b) for hydraulic conveying and intermittent flow at low particle loading.

Importance of viscous sublayer depth to particle velocity

Stevenson et al. (2001a,b) noticed that the relative size of the transported particle and viscous sublayer strongly determined the transport velocity of the particle in both intermittent flow and hydraulic conveying, respectively. Most previously work on the hydraulic conveying of particles (such as Turian et al., 1987) predicts a monotonic increase in the critical transport with particle size. Only Wilson (1979) suggests that there is a monotonic increase in critical velocity up to a certain particle size, whereafter the velocity required to transport particles (albeit at high solids holdup) decreases. In this work and in the related work on intermittent flow conveying and hydraulic conveying, it has been noticed that larger particles are transported with greater ease than smaller ones. This is thought to be due to the fact that larger particles are more exposed to coherent turbulent structures, because when at rest at the wall of the pipe, they protrude beyond the viscous sublayer into the buffer layer or even the turbulent core of the flow. When these turbulent events impinge on a particle, mobility is promoted. This effect can be re-expressed in terms of the eddy viscosity, which increases with distance from the wall. A small particle that resides within the viscous sublayer is subject only to molecular viscosity, larger particles are "gripped" by the fluid outside the sublayer which has much higher eddy viscosity.

It has been noticed that the average particle velocity in smooth stratified flow decreases as the liquid viscosity increases (Figure 3). This can also be explained by the depth of the viscous sublayer relative to the size of the particle. The viscous sublayer depth increases with liquid viscosity. Thus, the particle becomes more exposed to the buffer layer with its coherent turbulent structures as liquid viscosity decreases. This in turn aids particle mobility and so increases particle velocity.

Pipe inclination, particle shape and particle density

The greater effect of pipe inclination is contained in the increase in w with pipe declination at fixed j_f. Nonetheless, a slight further dependence of particle velocity on pipe inclination is discernible upon close inspection of Figure 4. It is of value to calculate the axial (with respect to the pipe) component of the submerged weight of the particle and compare it to an estimate of the drag force. This will determine whether it is the resolved submerged weight term in the di-

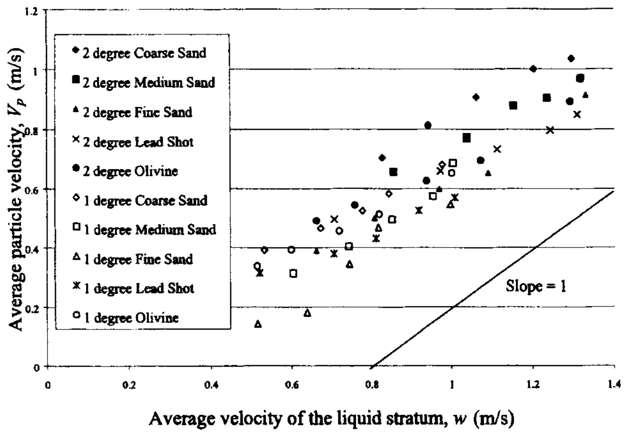

Figure 4. Average velocity of various particles in a 0.07 m pipe declined at 1° and 2° to the horizontal.

rection of flow that is causing the small further increase of velocity with greater pipe inclination or whether the increased pipe angle is affecting the physics of particle transport in a subtler manner. Assuming that the particles are perfect spheres of sieving diameter d, the axial (with respect to the pipe) component of the submerged weight W_θ is

$$W_\theta = \frac{\pi d^3}{6}\rho_f(s-1)g\sin\theta \tag{2}$$

where s is the ratio of particle to liquid density. The drag force is more difficult to calculate with accuracy. It may be estimated by considering the particle to be a perfect sphere travelling in a fluid moving at velocity w. The drag force may be then calculated based on the slip velocity $w - V_P$.

Take the example of the coarse sand in water flowing down a pipe declined at 2° to the horizontal. When the liquid velocity is 0.942 m/s, the average particle velocity was observed to be 0.813 m/s. The particle Reynolds number based on the slip velocity is 140.6. It is appropriate to calculate the drag coefficient that applies in the "intermediate" regime (Klinzing et al., 1997) where $C_D = 18.5/\mathrm{Re}_p^{0.6}$. Thus

$$F_D = \frac{18.5}{Re_p^{0.6}}0.5\rho_f(w-V_P)^2\frac{\pi d^2}{4} \tag{3}$$

Thus, the drag force is 7.4 μN and the axial component of the submerged weight is 0.36 μN (a factor of 20 smaller than the estimated drag force). It is reasonable to assert that the axial component of the weight of the particle is almost negligible when compared to the drag force for this example, especially when the scatter in the data is taken into account. Also, the liquid flow rate can only be measured by the rotameter to $\pm 5\%$. Moreover, because the weight is proportional to d^3 and the drag is approximately proportional to d^2, the weight component is still less significant for smaller grades of sand. However, at larger angles of pipe declination, the axial component of the submerged weight becomes more significant when compared to the drag force.

Clearly, the average particle velocity is dependent on pipe declination via two mechanisms. The most important is the average velocity of the liquid stratum (and, therefore, the drag force imparted on the particle by liquid flowing past) is dependent on pipe declination. The secondary dependence that only becomes significant at high pipe declination is the effect of the axial component of the particles submerged weight.

The authors know of no experimental data that have measured the velocity of an isolated grain saltating on a smooth pipe wall in hydraulic conveying other than that already discussed in this article. There is, however, a reasonably substantial body of data on the velocity of a single grain saltating on a sand bed. These data are summarized in Bridge and Dominic (1984).

Stevenson et al. (2001a) showed that the shape of a particle in hydraulic conveying strongly influenced the mobility. However, no systematic single parameter method of characterizing the effect that shape has on mobility has been determined. Moreover, in the current work, shape has not been varied independently of particle density. Therefore, in the first instance only the data for the three grades of sand (that have roughly the same angular shape) will be correlated in the next section.

The incorporation of a dependence on the ratio of particle density to liquid density s is also precluded in the first instance due to the fact that s has not been extensively varied independently of shape. However, because of the similarities noticed between the motion of particles in intermittent flow and hydraulic conveying, Stevenson et al. (2001a) suggested that the constant of gravitational acceleration g with $g(s-1)$. Their preliminary results on olivine suggested that this approach had some justification.

Correlation for velocity of an isolated particle in smooth stratified flow

It is reasonable to assert that, for sand of the same shape and density, the velocity of a single particle in stratified flow is independent on average liquid velocity, liquid viscosity, characteristic particle size, hydraulic diameter of the liquid stratum, and gravitational acceleration, that is

$$V_P = f(w, \nu, d, D_f, g) \tag{4}$$

Thus, Buckingham's π-Theorem suggests that three dimensionless groups are sufficient to characterize the system and a suitable set is

$$\frac{V_P}{w} = f\left(\frac{wd}{\nu}, \frac{gD_f}{w^2}\right) \tag{5}$$

If g is replaced by $g(s-1)$ for the reasons discussed above, the following results

$$\frac{V_P}{w} = f\left(\frac{wd}{\nu}, \frac{g(s-1)D_f}{w^2}\right) \tag{6}$$

On noticing that the data are linear in w (see Figure 4) and that an extrapolation of the data would cross the abscissa at a

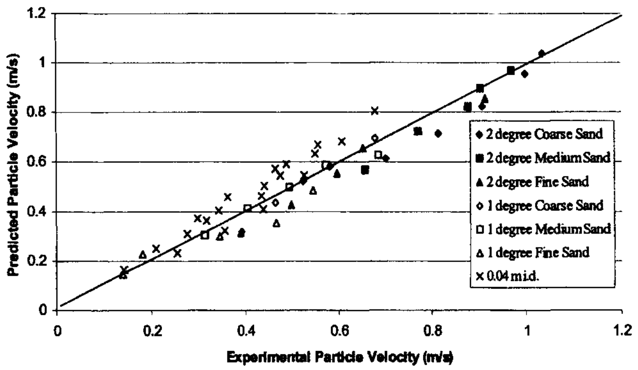

Figure 5. Success of correlation 7 demonstrated.
The data are for the 0.07 m ID pipe unless otherwise stated.

certain value of w, the "critical liquid velocity," the following correlation is developed using least-squares regression on the sand data set

$$\frac{V_P}{w} = 1.03\left[1 - 3.43\left(\frac{wd}{\nu}\right)^{-0.34}\left(\frac{g(s-1)D_f}{w^2}\right)^{0.33}\right] \quad (7)$$

Equation 7 shows that the critical liquid velocity w_C is independent of w.

Note that the slope of V_P vs. w is 1.03, that is, very close to the slope of one that was observed in both slug flow and hydraulic conveying by Stevenson et al. (2001a,b). The success of the correlation is shown in Figure 5. The RMS error of the predictions of velocity for sand particles is 0.009 m/s.

The correlation for the olivine and lead data is shown in Figure 6. The data for olivine, which is similar in shape, but denser than sand, closely match the predicted values giving further justification of the use of the $g(s-1)$ term in the Froude-type number in Eq. 7. However, the lead velocities are underpredicted by Eq. 7, which is exactly as is expected because the rounded shape of the lead shot is thought likely to enhance mobility. This result is complimentary to the conclusion of Stevenson et al. (2001b) for the behavior of lead shot in slug flow.

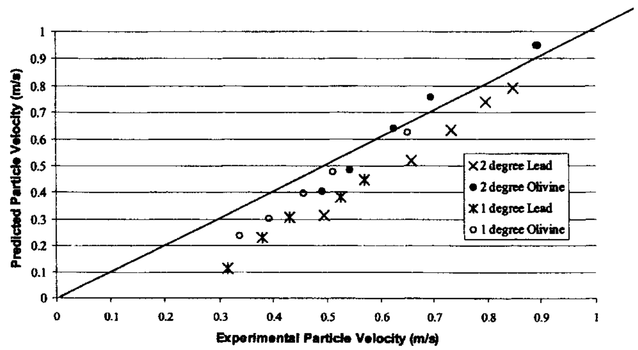

Figure 6. Average particle velocity predicted by correlation 7 for nonsand particles.

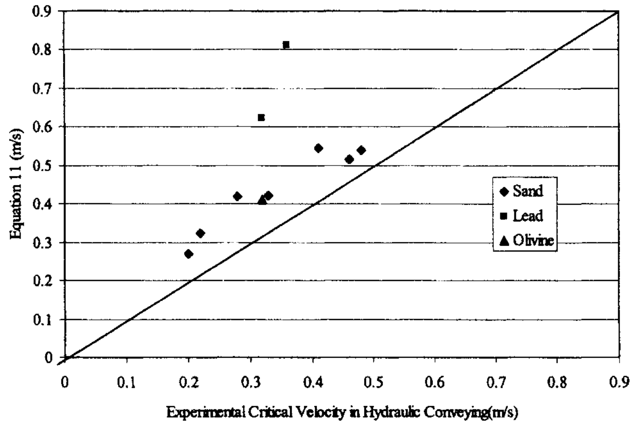

Figure 7. Predicted critical liquid velocity for particle transport in stratified flow against experimental data for hydraulic conveying.

Threshold of particle motion in stratified flow

The linear predictions of correlation 7 may be extrapolated to zero particle velocity to give an estimate of flow velocity required to ensure particle movement w_C.

$$w_C = 3.43 d^{-0.34} \nu^{0.34} \left[g(s-1)D_f\right]^{0.33} \quad (8)$$

The predictions of critical velocity are compared with those experimentally observed in hydraulic conveying by Stevenson et al. 2001b (Figure 7). All data are for sand in the 0.07 m pipe at various viscosities; data for lead shot and olivine are specifically marked.

The predictions of critical velocity are slightly greater than those observed in hydraulic conveying. However, if one considers that the predictions are obtained by extrapolating a data set and are then compared with a different flow regime altogether, the agreement is striking. However, extrapolation of Eq. 7 to zero particle velocity greatly overpredicts the critical velocity for the transport of lead shot by hydraulic conveying due to its rounded shape. This is complimentary to the graph produced by Stevenson et al. (2001a) who extrapolated to zero the correlation that they had developed for the velocity of isolated particles in horizontal slug flow. They found that the critical velocities in hydraulic conveying were recovered for angular particles (that is, sand and olivine), but the critical velocity for lead shot was overpredicted due to the shape of the lead shot that made it easier to move.

Conclusions

Experiments to measure the velocity of particles of sand, olivine, and lead shot in slightly declined smooth stratified gas/liquid flow have been performed. Particle velocity is linear in the average velocity of the liquid stratum.

It is seen that large particles travel more quickly than smaller ones, and that particle velocity decreases with liquid viscosity. This is explained by consideration of the particle size relative to the depth of the viscous sublayer.

A correlation is presented to predict particle velocity in smooth stratified flow. This correlation underpredicts the velocity of lead shot, because the rounded shape of the lead enhances mobility compared to the angular shape of the sand and the olivine.

The data for particle velocity against the average velocity of the liquid stratum are linear with a gradient of approximately one. This is comparable to the results of Stevenson et al. (2001a) for hydraulic conveying where particle velocity is seen to be linear with respect to liquid velocity, with a gradient of one, and with the results of Stevenson et al. (2001b) for intermittent flow, where particle velocity is seen to be linear with respect to the velocity of fluid in the slug body, with a gradient of approximately one. The predictions of the correlation are extrapolated to zero particle velocity in order to give an estimate of the average velocity in the liquid stratum required to ensure particle transport (the critical velocity). The estimated critical velocity for particles in smooth stratified flow is in good agreement with the critical velocity of particles observed in hydraulic conveying.

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Notation

A = cross-sectional area, m^2
C = circularity of a 2-D projection of a particle defined in Eq. 1
C_f = Fanning friction factor
C_D = particle drag coefficient
d = Sieving diameter of particle, m
D = Internal diameter of the pipe, m
F_D = Drag force imparted on the particle, N
g = acceleration due to gravity, $m \cdot s^{-2}$
j = superficial velocity
p = perimeter of a 2-D projection of a particle, m
p_e = perimeter of an equivalent circle, m
Re = Reynolds number based on D
Re_P = Particle Reynolds number
s = ratio of particle to liquid density
u = absolute velocity, $m \cdot s^{-1}$
V_P = average particle velocity, $m \cdot s^{-1}$
w = average velocity of the liquid stratum, $m \cdot s^{-1}$
W_θ = axial component of particle weight with respect to the pipe, N
w_C = critical velocity of the liquid stratum, $m \cdot s^{-1}$
θ = angle of pipe declination
ν = kinematic viscosity, $m^2 \cdot s^{-1}$
ρ = density, $kg \cdot m^{-3}$
τ_i = interfacial shear stress, $N \cdot m^{-2}$
τ_W = wall shear stress, $N \cdot m^{-2}$

Subscripts

f = liquid phase
g = gas phase
i = interfacial

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